## Exercise 21

In Exercises 17–24, find the unknown if the solution of each equation is given:

$$\text{If } u(x) = e^x \text{ is a solution of } u(x) = f(x) + \int_0^x \left(2u^2(t) + u(t)\right) dt, \text{ find } f(x)$$

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**Solution**

Substitute the solution into both sides of the equation.

$$
\begin{aligned}
e^x &= f(x) + \int_0^x \left(2e^{2t} + e^t\right) dt \\
&= f(x) + \left(\int_0^x 2e^{2t}\, dt + \int_0^x e^t\, dt\right) \\
&= f(x) + e^{2t}\Big|_0^x + e^t\Big|_0^x \\
&= f(x) + e^{2x} - e^0 + e^x - e^0 \\
&= f(x) + e^{2x} + e^x - 2
\end{aligned}
$$

Therefore,

$$f(x) = 2 - e^{2x}.$$